HOW I TEACH NUMBER PATTERNS

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This 'How I Teach' presentation demonstrates how two consecutive topics from the Grade 8 CAPS schedule for Grade 8, namely Number and Geometric Patterns *and* Functions and Relations *were compressed into one set of seven lessons.*

INTRODUCTION

One of the challenges of high school mathematics is getting to grips with the abstract concepts presented in Patterns, Functions and Algebra. This is the first year that I experience having to pace my teaching according to CAPS. Only a week is allowed for patterns. Using an extra day for patterns will result in falling behind with the required pace that I should be working at. Bear in mind that the same is true with every other topic in mathematics. When preparing for the set of lessons, I considered this quote from the Senior Phase CAPS document:

In Patterns, Functions and Algebra, learners' conceptual development progresses from … the recognition of patterns and relationships, to the recognition of functions, where functions have unique output values for specified input values.

(Senior Phase Mathematics Curriculum and Assessment Policy Statement, 2011, page 21)

Rationale

I realized that number patterns provide an opportunity to lead learners to the point where they recognise functions based on the relationship between the input and output values. Topic 2.2, *Functions and Relationships* states that learners should "determine input or output values or rules or patterns and relationships using flow diagrams, tables, formula, equations" (2011: 22). In Term 1 the topics *Numeric and Geometric Patterns* and *Functions and Relationships* follow one another. There are 4,5 hours and 3 hours allocated to the two topics respectively. I decided to merge the content with the aim of helping learners to recognise functions where functions have unique output values for specified input values.

Within the topic *Numeric and Geometric Patterns* Grade 7 and 8 learners should investigate and extend numeric and geometric patterns and represent their solutions in tables. By Grade 8 they are expected to also represent the rule algebraically and the range of number patterns is extended to include patterns with multiplication and division of integers and numbers in exponential form. This provides an opportunity for learners to engage with basic number concepts.

I also consider it important to make the distinction between 'position of the term' and 'term' in a sequence. This prepares the way for introducing the terminology 'input-' and 'output-value'. Investigating number patterns is an opportunity to generalize, to give general algebraic descriptions of the relationship between terms and their position in a sequence and to justify solutions.

For the topic Functions and Relationships, learners are expected to determine input and output values or rules for patterns and relationships using flow diagrams, tables, formulae and equations, with the latter being the only extension from Grade 7 to 8. They are also expected to recognise the equivalence between the different representations.

CONTENT

By combining the two topics I afford learners 7,5 hours to understand these two related concepts. This amounts to approximately 7 lessons. The content of the lessons is outlined below.

Day 1: Investigating and Extending Patterns
I started with a few revision exercises just to refresh the memory of the learners. The exercise below is an example of work covered.

Consider the following sequence:

    4 ; 7 ; 10 ; 13

or

Consider the pattern

Give the difference between the consecutive terms.

Describe the rule you found in words.

What would the values of the next 3 terms be?

What would the value of the 10th term be?

As mentioned earlier, for the learners to understand these questions, they would have to know the definition of rule and term. It is therefore important to revise these terms before giving these exercises. The first day could basically be spent on identifying patterns with common differences and ratios, and describing the relationship between terms and the term number. This should be a familiar task as learners covered this ground in Grade 7.

Day 2: Getting learners to
- **extend geometric and numeric patterns**
- **complete the table representing the pattern, by filling in the missing output values**

Learners would be expected to extend a pattern in the table, where some terms are missing. They would be expected to find the general rule. This is also a revision task, but many learners may struggle.

Day 3

Same as Day 2, but including a flow diagram where learners will fill in the operations determined by the general rule.

This task takes learners to a level where, besides determining the general rule, they are expected to break up the rule into its different operations in their correct order. This forces them to consider the order of operations, an important inclusion in Topic 1: Number, Operations and Relations. I include excerpts from a Day 3 task:

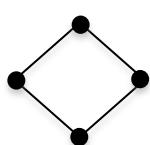Learners were given a pattern like the one below:

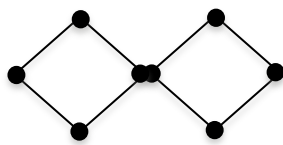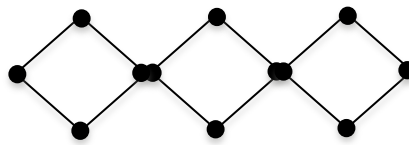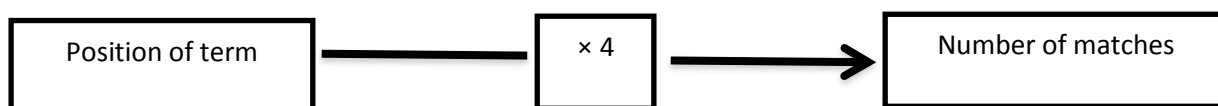FIGURE 1   FIGURE 2   FIGURE 3

After doing the basic tasks of extending the pattern and completing a simple table by filling in missing output values, most were able to see that the rule was "multiply the term number by 4". They were then given this flow diagram:


| Position of term | → | × 4 | → | Number of matches |

The value of this flow diagram is that it highlights the relationship between the input value (position of the term) and the output value (term i.e. number of matches). Learners would be challenged to complete a table based on the rule represented in the flow diagram.

Position of term (n)	8	12	15	22	31	50
Number of matches (m)						

Day 4

Learners continue to work with flow diagrams, most representing rules comprising multiple operations. For example, the geometric pattern that yields the rule $m = 3n - 2$ is presented to learners, who are expected to fill in the missing outputs and input:
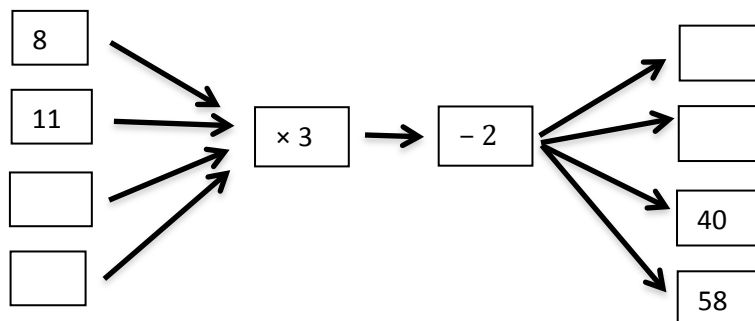
To derive the missing input, learners will realize that they need to apply inverse operations to those indicated in the flow diagram. This is a powerful way to introduce inverse operations.

They would also be expected to write the rule as an algebraic equation, and to complete a table.

Days 5, 6 & 7

Learners consolidate their understanding of number patterns and are expected to apply inverse operations to derive input values from given outputs.. By Day 6 I introduce the notion of constant difference and constant ratio, and look at functions where there is neither constant difference nor ratio. I show them a systematic method to derive the rule for a number pattern.

CONCLUSION

Using this strategy has many advantages. In the past, learners really struggled to find the nth term. This method underscores the notion of functional relationships between the input and output value and helps them to derive the rule, which is generalized as the equation containing the nth term. When moving on to algebraic equations later in the year, it will also aid learners to understand the concept of applying the inverse operations to solve for x and using the "check" method to prove that the answer is true.

REFERENCES

Department of Basic Education, 2011. National Curriculum Statement; Curriculum and Assessment Policy Statement., Mathematics, Grade 7 – 9. Department of Basic Education, Pretoria.

Lewis, A. et al, 2012. Mathematics Companion Learner's Workbook Grade 8 Volume 2. The Answer Series, Claremont.